# Weighted Piece-wise Linear Interpolation (WPLI)

Linear interpolation assumes the target function to be a straight line between any two consecutive points. Mathematically, in a two-dimensional plane, if two points () and () are given, then any points located in the interval [] lies in the straight line connecting () and (). The value of such a point can be found using the following equation:

(3)

In this study, each represents a date between May 26th, 2024, to July 7th, 2024. is a spatial ET value of pixel for the corresponding date. Using the daily ETc readings from the in-situ canopy sensors, we can extract a trend from those values instead of assuming a straight line between two consecutive missions. The trend will result in more accurate ET maps for each day and show the increase and decrease of ET values corresponding to the change in meteorological parameters. To account for this trend, linear interpolation technique was modified and we developed a modified method and called it as Weighted Piece-wise Linear Interpolation (WPLI). WPLI method basically observes the daily sensor-based ETc trends as a piece-wise linear line. Then it models the daily pixel-specific ET proportionally based upon the predecessor and the successor aerial mission—where the importance of each mission is weighted as per the date-difference between their operating dates and the target date of estimation.

The readings of the infield sensor is to find the reference trend—denoted by . Then, for each pixel in the map, the following procedures are followed:

* For two consecutive missions, and , calculate the ratios and of the ET values in the map and the Arable sensor.

(4)

where

* Find the forward trend , using .

(5)

where

* Find the backward trend , using .

(6)

where

* For each day , approximate the ET value from and using the following equations.

(7)

(8)

Eqn. 8 uses the concept of Inverse distance weighting (IDW) on time scale. In IDW, the value of an unknown point is calculated based on its distance from two or more known points. A weight factor was added on each known point such that the factor decreases with the increase in distance. We adopted the same approach for time series data. For a non-mission day , the weight of would be proportionally greater than the weight of in calculating . Iterating using this approach in between the consecutive missions and , the resultant vector is a piece-wise linear graph.

# Forward Interpolation

Forward Interpolation (FI) uses a similar concept to WPLI. However, in this method, the value of any pixel in the spatial ET image is estimated from the previous mission(s) only. The goal of this method is to predict the ET based on ET-ETc ratios (s from eqn. 4) from the previous mission(s) and the ETc on the target day. In other words, we are projecting past trends into the future and using this trend to predict spatial ET for each individual pixel using the concepts of linear interpolation. In doing this, the equations of IDW are modified to be unidirectional in the time-series. That means, in the time scale, all the known data points are in the past (in one direction only) compared to the unknown data. The procedures for this method to approximate each pixel are as follows:

* Calculate for each (the mission on date and all its previous missions of the season) using eqn. 4. For example, if we want to find the spatial ET for July 1st, 2024 (a date with no mission), would include the missions on May 26th, June 2nd, June 9th, June 16th, June 23rd, and June 30th, 2024, respectively (here, refers to May 26th, 2024, and refers to June 30th, 2024).
* Find separate forward trends using each obtained , using eqn. 5 in the interval (similar notation as eqn. 5). In practice, can be any day in the future, but here we assumed that is the next scheduled mission for evaluation purposes (see more details in the results section).
* For each day , calculate the corresponding weight factor of each mission using the following equations.

(9)

(10)

As shown in eqns. 9 and 10, for each day, the weight factor for each previous mission is inversely proportional to the difference between the target day and the corresponding mission (the difference of the days must be at least 1—which is why we used an open interval at the beginning). The obtained weights are then normalized in the following way.

(11)

Approximate the ET value using all forward trends () and their corresponding weight factors () using the following equation.

(12)

Eqn. 12 shows that the closer the missions from the target date, the more weights are assigned to the corresponding mission data.

We adapted three variations of FI method named FI-1, FI-2, and FI-∞ respectively. As the name suggests, FI-1 considers only one previous mission, FI-2 considers the previous two missions and FI-∞ considers all the previous missions of the season while approximating the ET for any non-mission day. In a similar fashion, any variations of the FI method can be modelled (for example, FI-k, where k = 1, 2, 3, 4…). The reason for adapting these variations was to see if the value of ET for a specific date depends on the most recent field condition, or all the past days since the beginning of the season.